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Procedural Utility and Cooperation: A Simple Illustration

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Department of Economics, Hannam University, 70 Hannamro, Daedeok Gu, Daejeon 306-791 Republic of Korea Abstract. The current research shows that a cooperative equilibrium in prisoners' dilemma can be explained by incorporating procedural utility into a traditional utility function. This study contributes to the existing literature by showing that cooperation can be explained by a simple model that is not radically different from the traditional model. Keywords: Procedural Utility, Cooperation, Prisoners' Dilemma

1 Introduction

For traditional economic theory, utility is defined over a restricted set of outcomes. Various critiques have been made about the outcome-focused concept [see 6, 16, 18]. Recent works of Frey and colleagues have contributed to the development of an alternative concept of utility by proposing procedural utility. Frey [7, p.109] defines procedural utility as "the well-being gained from living and acting under institutionalized processes" that address the innate needs for self-determination. Procedural utility theory starts from the recognition that utility comes not only from actual outcomes, but also from the procedures that lead to the outcomes. Frey and colleagues examine procedural utility in various contexts, including the allocation of public goods, government redistribution of income, the handling of arbitration cases, and individuals' participation in the democratic process, and they highlight past empirical evidence that individuals derive procedural utility in many areas [for surveys, see 1, 11].

Frey and Stutzer [10] discuss the three sources of procedural utility: Institutions, behaviors, and interactions. First, people care about how they are treated by institutions because this has an impact on how they feel about themselves. For example, people derive procedural utility from democracy, because it provides a feeling of being involved and having political influence, as well as a notion of inclusion, identity and self-determination [8, 9]. Another example is the empirical evidence of being self-employed as a source of procedural utility [2, 3]. In contrast to hierarchy (e.g., the firm with some degree of authority), the market (e.g., being self-employed characterized by independent decisions) leads to a higher self-determination. Second, people get satisfaction from a particular act "when they have an intrinsic attitude towards the action or choice process they are involved in" [10, p.92]. For example, people might derive utility from the mere act of gambling or donation. Third, people have utility for acting in a fair way or being treated fairly. For example, Benz and Stutzer [4] find that worker satisfaction with pay is significantly influenced by procedural factors such as the frequency of being asked about pay issues by superiors.

It is widely known that Nash equilibrium in prisoners' dilemma is a 'defection' solution that is

not a socially best outcome, even though individual players are rational in the sense that they are concerned only with maximizing their own payoffs. Although game theory identifies defection as the only Nash Equilibrium in a one-shot prisoners' dilemma setting, both cooperative and defective behaviors occur in real life as well as in laboratory experiments. This problem has been around for a long time, and there have been attempts to reconcile rational behavior with cooperation [for earlier literature, see 12, 13, 17]. A common approach to explaining cooperation is to consider repetitions of the game. In the repeated game setting of prisoners' dilemma, as long as the probability of game repetition is sufficiently high and individuals are sufficiently patient, a cooperative equilibrium can be sustained. This is a so-called Folk Theorem for repeated games.

Procedural utility also matters for cooperation. Le Menestrel [14] proposes a model of game theory by taking procedural utility into account and shows that, in the model, a cooperative equilibrium can be reached in a one-shot prisoners' dilemma game. This study proposes a similar model of game theory and analyzes one-shot, repeated, and evolutionary games. This illustration demonstrates that a Nash equilibrium can be more cooperative than expected by traditional game theory. While the modelling of this paper is an adaptation of Le Menestrel [14], the novelty here resides in the formulation about repeated games and evolutionary games.

Note also that the utility function developed in the current study is not conceptually different from the traditional utility function. Lee [15] explains that there are few theoretical differences between the procedural utility model and traditional utility model. Thus, it would show that analyzing cooperative solutions does not require radical changes in setting up a basic model. Cooperation can be explained by using a simple model that is similar to a standard model.

2 A One-Shot Game

Consider the strategic form of an *N*-player game. Let $S = (s_1, \ldots, s_n)$ be the set of pure strategies. Player *i* can take an available pure strategy $s_i \in S$ that leads to a particular payoff, $\pi_i = \pi_i(s_i, s_{-i})$, where $s_{-i} = \{s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N\}$.

According to the procedural utility theory, players derive utility from the payoff, π_i , as well as the strategies, s_i and s_{-i} , taken to produce the payoff. We assume for simplicity that players have separable utility defined over strategies and over outcomes. The rule of the game determines the utility function for each player *i* as follows:

$$u_{i} = u_{i}(s_{i}, s_{-i}, \pi_{i}(s_{i}, s_{-i}))$$

= $f_{i}(s_{i}, s_{-i}) + g_{i}(\pi_{i}(s_{i}, s_{-i}))$ (1)

where $f_i(\cdot)$ is the procedural utility function and $g_i(\cdot)$ is the traditional (outcome-focused) utility function with $g'_i(\pi) > 0$.

The procedural utility function, $f_i(\cdot)$, implies that procedural utility is dependent upon the strategy taken by the player *i*, s_i , as well as the strategies taken by the other players, s_{-i} . This

implies the three sources of procedural utility discussed by Frey and Stutzer [10]. The first source, particular institutions, is related to the strategies taken by the other players since they become the environment faced by the player i. The second one, a particular action, is just the strategy taken by the player i. Players might have preferences for certain actions, that is, strategies. The third one, an interaction between people, include both of the strategy of player i and the strategies of the other players.

Generally, a Nash equilibrium is defined as follows:

Definition 1. A strategy profile $\{s_1^*, s_2^*, \ldots, s_N^*\}$ of the N-player game is a Nash equilibrium if

$$f_i(s_i^*, s_{-i}^*) + g_i(\pi_i(s_i^*, s_{-i}^*)) \ge f_i(s_i, s_{-i}^*) + g_i(\pi_i(s_i, s_{-i}^*))$$

$$(2)$$

for all $s_i \in S$ and all $i \in N$.

This Nash equilibrium is composed of four cases. First, the utility level of the Nash equilibrium is higher than any other level of utility in terms of both procedural utility and outcome-focused utility. Put differently, the player is satisfied by the outcome of the Nash equilibrium as well as the process that leads to the outcome. Second, while the procedural utility level of the Nash equilibrium is higher than that of the other strategy profiles, the outcome-focused utility level of the Nash equilibrium is not the highest one. The player is satisfied with the process, but not with the outcome. In this case, the benefit of procedural utility should dominate the cost of outcome-focused utility in order for this to be a Nash equilibrium. The third is the inverse of the second case. The benefit in terms of outcome-focused utility outweighs the cost in terms of procedural utility in this case. Finally, although each utility is not the highest in each category, the aggregation leads to the Nash equilibrium. The four cases can be expressed as follows:

Case 1
$$f_i(s_i^*, s_{-i}^*) \ge f_i(s_i, s_{-i}^*)$$
 and $g_i(\pi_i(s_i^*, s_{-i}^*)) \ge g_i(\pi_i(s_i, s_{-i}^*))$ (3)

Case 2
$$f_i(s_i^*, s_{-i}^*) \ge f_i(s_i, s_{-i}^*), \quad g_i(\pi_i(s_i^*, s_{-i}^*)) \le g_i(\pi_i(s_i, s_{-i}^*))$$

and
$$f_i(s_i^*, s_{-i}^*) - f_i(s_i, s_{-i}^*) \ge g_i(\pi_i(s_i, s_{-i}^*)) - g_i(\pi_i(s_i^*, s_{-i}^*))$$
 (4)

Case 3
$$f_i(s_i^*, s_{-i}^*) \le f_i(s_i, s_{-i}^*), \quad g_i(\pi_i(s_i^*, s_{-i}^*)) \ge g_i(\pi_i(s_i, s_{-i}^*))$$

and $a_i(\pi_i(s_i^*, s_{-i}^*)) = a_i(\pi_i(s_i^*, s_{-i}^*)) \ge f_i(s_i^*, s_{-i}^*) = f_i(s_i^*, s_{-i}^*)$ (5)

Case 4
$$f_i(s_i^*, s_{-i}^*) \le f_i(s_i, s_{-i}^*), \quad g_i(\pi_i(s_i^*, s_{-i}^*)) \le g_i(\pi_i(s_i, s_{-i}^*))$$

and
$$f_i(s_i^*, s_{-i}^*) + g_i(\pi_i(s_i^*, s_{-i}^*)) \ge f_i(s_i, s_{-i}^*) + g_i(\pi_i(s_i, s_{-i}^*))$$
 (6)

Now we will consider a two-strategy two-player prisoners' dilemma game. The two strategies are cooperation (C) and defection (D). Suppose that $a = g_i(\pi_i(C,C))$, $b = g_i(\pi_i(D,D))$, $c = g_i(\pi_i(C,D))$, and $d = g_i(\pi_i(D,C))$ for all *i*. In this case, the condition for setting up the prisoners' dilemma is d > a > b > c. The payoff matrix 7 shows the strategic form of the traditional model of prisoners' dilemma that deals with the outcome-focused utility only:

$$\begin{array}{c|c}
C & D \\
C & a, a & c, d \\
D & d, c & b, b
\end{array}$$
(7)

It is widely known that, in the traditional model, the strategy profile (D, D) is a Nash equilibrium and the players will get the payoffs (b, b) although the higher set of payoffs they can attain is (a, a).

The payoff matrix can be modified if we introduce the procedural utility into the traditional utility function. Suppose that $f_{cc} = f_i(C,C)$, $f_{cd} = f_i(C,D)$, $f_{dc} = f_i(D,C)$, and $f_{dd} = f_i(D,D)$ for all *i*. Now the payoff matrix with procedural utility is:

$$\begin{array}{c|c} C & D \\ \hline C & f_{cc} + a, f_{cc} + a & f_{cd} + c, f_{dc} + d \\ D & f_{dc} + d, f_{cd} + c & f_{dd} + b, f_{dd} + b \end{array}$$
(8)

If we consider procedural utility, the cooperative solution (C, C) can emerge as a Nash equilibrium.

Theorem 1. The cooperative outcome (C, C) can be a Nash Equilibrium of the one-shot prisoners' dilemma with procedural utility if players sufficiently prefer cooperation to defection in an atmosphere of cooperation.

Proof. The strategy profile (C, C) is a Nash equilibrium if and only if

$$f_{cc} + a \ge f_{dc} + d. \tag{9}$$

The condition 9 corresponds to **Case 2** because d > a. This condition can be rewritten as

$$f_{cc} - f_{dc} \ge d - a. \tag{10}$$

 f_{cc} refers to the level of procedural utility when a player *i* cooperates and the opponent cooperates, and f_{dc} refers to the level of procedural utility when a player *i* defects and the opponent cooperates. Therefore, the condition 10 implies that if players much prefer cooperation to defection in an atmosphere of cooperation (that is, when the other player cooperates) such that the benefit in procedural utility ($f_{cc} - f_{dc}$) is large enough to dominate the cost in outcome-focused utility (d - a), mutual cooperation can be obtained.

Note that $f_{cc} + a$ is not different qualitatively from a. Including procedural utility into traditional utility function does not make the model something radically different from the traditional model. Thus, what matters in the model is whether people will prefer cooperation if their partner cooperates in a one-shot game, and whether procedural utility derived from mutual cooperation will be sufficient to make the cooperative outcome an equilibrium. Berg et al. [5] provide insightful evidence that trust and reciprocity can occur even in a one-shot setting and this can be reinforced by social history.

3 A Repeated Game

The Folk theorem shows that, for repeated prisoners' dilemma games, a cooperative solution can be sustained as an equilibrium if players are sufficiently patient. In this section, we show that procedural utility increases the possibility of a cooperative equilibrium in a prisoners' dilemma game.

Let us refer to the payoff matrix 8 in which procedural utility is included. Suppose the players play the game at times $t = 0, 1, 2, ..., \infty$ and the payoff to each player will be the present value of the payoff stream over all time periods, calculated using a discount factor δ , with $0 < \delta = \frac{1}{1+i} < 1$, where i > 0 is the corresponding discount rate.

Theorem 2. Suppose that each player uses the trigger strategy of cooperating as long as the other player cooperates, and defecting forever if the other player defects. The cooperative outcome (C, C) is more likely to be a Nash equilibrium of a repeated prisoners' dilemma with procedural utility if players sufficiently prefer an atmosphere of cooperation (C, C) to an atmosphere of defection (D, D).

Proof. If both players stick to the strategy of cooperation, the payoff to each player will be the discounted sum of the payoffs,

$$\sum_{t=0}^{\infty} (f_{cc} + a)\delta^t = \frac{f_{cc} + a}{1 - \delta}.$$
(11)

If one player defects once, the best response to the player is to defect forever thereafter because the other player will defect forever. If we assume that one player defects on the first round of play, the payoff in this situation will be

$$f_{dc} + d + \sum_{t=1}^{\infty} (f_{dd} + b)\delta^t = f_{dc} + d + (f_{dd} + b)\frac{\delta}{1 - \delta}.$$
 (12)

Thus, the cooperative outcome is a Nash equilibrium if and only if

$$\frac{f_{cc} + a}{1 - \delta} \ge f_{dc} + d + (f_{dd} + b)\frac{\delta}{1 - \delta}$$
(13)

The condition 13 can be reorganized as follows:

$$\delta \ge \frac{f_{dc} - f_{cc} + d - a}{f_{dc} - f_{dd} + d - b}.$$
(14)

If all procedural utilities are set to zero, one obtains the traditional result without procedural utility.

$$\delta \ge \frac{d-a}{d-b}.\tag{15}$$

The condition 15 implies the Folk Theorem that, if the discount factor, δ , is sufficiently large, the

condition for mutual cooperation to be a Nash equilibrium is met although the one-shot payoff of cooperation, a, is not much larger than the one-shot payoff of defection, b.

What happens to the condition for cooperation if procedural utility is taken into account? The condition 14 indicates that, if f_{cc} is much higher than f_{dd} , then the right hand term becomes small, so that the condition for mutual cooperation to be an Nash equilibrium is more likely to be met although δ is not sufficiently large.

If we consider procedural utility, we can have a less restrictive condition for mutual cooperation to be a Nash equilibrium of a repeated prisoners' dilemma game. In the world of procedural utility, the cooperative solution can be a Nash equilibrium even though people much prefer present time over future. In this sense, procedural utility derived from cooperation can be a substitute for preferences for present time.

4 An Evolutionary Game

In an evolutionary game, a player is assumed to be genetically and culturally programmed for playing a particular strategy. That is, each player is of type i for some strategy $s_i \in S$ and the player of type i is supposed to employ the strategy s_i in the game. Let p_i be the proportion of players playing s_i . Then, the state of the population is defined as

$$\sigma = \sum_{i=1}^{n} p_i s_i,\tag{16}$$

and the payoff to a player of type i is

$$\pi(s_i) = \sum_{j=1}^n \pi(s_i s_j) p_j,$$
(17)

where $\pi(s_i s_j)$ is the payoff to the player of type *i* when the opponent plays s_j . If we replace a fraction $\epsilon > 0$ of the population with a group of players of type *j*, then the new state of the population will be

$$\tau = (1 - \epsilon)\sigma + \epsilon s_j. \tag{18}$$

Now we can calculate the expected outcome to a randomly chosen player of each population in each state. The expected outcome to a randomly chosen player of the incumbent population in the initial state is

$$\pi(\sigma\sigma) = \sum_{i=1}^{n} \pi(s_i) p_i,$$
(19)

the expected outcome of the incumbent population in the new state is

$$\pi(\sigma\tau) = (1 - \epsilon)\pi(\sigma\sigma) + \epsilon\pi(\sigma s_j),\tag{20}$$

the expected outcome of the new population in the initial state is

$$\pi(\tau\sigma) = (1-\epsilon)\pi(\sigma\sigma) + \epsilon\pi(s_i\sigma), \tag{21}$$

and the expected outcome of the new population in the new state is

$$\pi(\tau\tau) = (1-\epsilon)\pi(\sigma\tau) + \epsilon\pi(s_j\tau).$$
(22)

An evolutionarily stable equilibrium is defined as follows:

Definition 2. The state of the population, σ , is an evolutionarily stable equilibrium if and only if, for any other state τ ,

$$\pi(\sigma\sigma) > \pi(\tau\sigma) \tag{23}$$

or if $\pi(\sigma\sigma) = \pi(\tau\sigma)$, then

$$\pi(\sigma\tau) > \pi(\tau\tau). \tag{24}$$

The state of the population with a set of strategies is an evolutionarily stable equilibrium if (i) no one playing one strategy could improve its outcome by switching to one of the other strategies in the state of the population, and (ii) no one playing a different strategy altogether could establish itself in the state of the population.

An evolutionarily stable equilibrium is a Nash equilibrium, since, if σ is evolutionarily stable, then the condition $\pi(\sigma\sigma) \geq \pi(\tau\sigma)$ should be met, which is just the condition for the Nash equilibrium. Note that an evolutionarily stable equilibrium is a Nash equilibrium, but not the other way around. What about the traditional Nash equilibrium in prisoners' dilemma games? In the traditional prisoners' dilemma setting, (D, D), is an evolutionarily stable equilibrium because, it is obvious that, in the state in which all players are of type D, anyone of type C cannot invade the population. On the contrary, the cooperative solution, (C, C), is not evolutionarily stable, since it is not a Nash equilibrium. However, things change if procedural utility steps in. Refer to the procedural utility payoff matrix 8.

Theorem 3. The cooperative solution of the prisoners' dilemma game, (C, C) in the two-player game, can be an evolutionarily stable equilibrium when we consider procedural utility.

Proof. According to Theorem 1, the cooperative solution can be a Nash equilibrium, and thus we only need to check the evolutionary stability of the cooperation. Suppose that the state of the population is

$$\sigma = 1 \cdot C \tag{25}$$

and the new state of the population is

$$\tau = (1 - \epsilon)C + \epsilon D. \tag{26}$$

The condition for the cooperative solution 25 to be evolutionarily stable is

$$f_{cc} + a \ge (1 - \epsilon)(f_{cc} + a) + \epsilon(f_{dc} + d).$$

$$(27)$$

Note that the condition 27 is the same as the condition 9 discussed above. If the equality holds in 27, we need to check the second condition 24:

$$(1-\epsilon)(f_{cc}+a) + \epsilon(f_{cd}+c) > (1-\epsilon)^2(f_{cc}+a) + \epsilon(1-\epsilon)(f_{dc}+d) + (1-\epsilon)\epsilon(f_{cd}+c) + \epsilon^2(f_{dd}+b).$$

$$(28)$$

With a sufficiently small value for ϵ , the condition 28 can be met.

We can confirm Theorem 3 by using replicator dynamics. Replicator dynamics in evolutionary games implies that the fraction of the players of certain type will increase as time passes if it performs better than the average, otherwise, it will decrease. It can be represented by a first-order differential equation:

$$\frac{dp}{dt} - p[\pi - \overline{\pi}] = 0, \tag{29}$$

where $\overline{\pi}$ is the average payoff of the state of the population.

Refer to the payoff matrix 7 of the traditional prisoners' dilemma. Let p be the proportion of players of type D and $\pi = \pi(D)$. The first-order differential equation is

$$f(p) = \frac{dp}{dt} = p(Ap - B)(1 - p),$$
(30)

where A = a + b - c - d and B = a - d. The phase diagram 1 represents the differential equation 30.



Figure 1: Phase Diagram of the Replicator Dynamics

The replicator dynamics has two steady states: p = 0 and p = 1. $\frac{B}{A}$ cannot be a proportion since it should be more than 1 or less than 0. Among the two steady states, the point p = 0 cannot be a stable equilibrium since, above as well as below that point, the dynamics consistently lead away from the point. Therefore, only one stable equilibrium exists in the replicator dynamics: p = 1. We can obtain the same result by using the condition for a stable equilibrium, f'(0) > 0 and f'(1) < 0. The traditional Nash equilibrium, p = 1, is evolutionarily stable since it is a stable equilibrium in replicator dynamics. In the population, everybody will have the same type D. We can also use the replicator dynamics to prove that the cooperative solution is not an evolutionarily stable equilibrium in the traditional model since, for type C, p = 0 is only one stable equilibrium.

Now refer to the payoff matrix 8 of the prisoners' dilemma with procedural utility. Let p be the proportion of players of type C and $\pi = \pi(C)$. The first-order differential equation is

$$f(p) = \frac{dp}{dt} = p(Ap - B)(1 - p)$$
(31)

where $A = (a + f_{cc}) - (c + f_{cd}) - (d + f_{dc}) + (b + f_{dd})$ and $B = (c + f_{cd}) - (b + f_{dd})$. The condition for the cooperative solution to be an evolutionarily stable equilibrium is

$$f'(0) = -B > 0$$
 and $f'(1) = -A + B < 0.$ (32)

First, *B* is negative when the sum of procedural and outcome-focused utilities derived from (C, D) is less than the sum of the utilities derived from (D, D), which can make sense in the context. Second, *A* can be greater than *B* if f_{cc} is much greater than f_{dc} . Therefore, the cooperative solution can be an evolutionarily stable equilibrium if procedural utility is sufficiently important.

5 Conclusion

The analysis shows that incorporating procedural utility into game theory can help explain a cooperative equilibrium. First, in a one-shot prisoners' dilemma game, given that people derive procedural utility from a cooperative atmosphere, a cooperative equilibrium can be attained. Second, cooperation in a repeated prisoners' dilemma setting is more likely to emerge in the case that procedural utility exists. Third, a cooperative solution can be an evolutionarily stable equilibrium under the presence of procedural utility.

This study contributes to the existing literature by showing that cooperation can be explained by a simple model that is not essentially different from the traditional model. We do not need to set up a radically different model of game theory. A simple model that is similar to a traditional model is sufficient to explain cooperation in a prisoners' dilemma game.

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